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Free Vibration Analysis of Simply Supported Power Law Functionally Graded Beam Using Finite Element Method

Zainab Abbood[1,a,*] & Luay S. AlAnsari[1,b]

[1] Department of Mechanical Engineering, University of Kufa, Najaf, Iraq
[a]Zainababboud1994@gmail.com
[b]luays.alansari@uokufa.edu.iq
[*]Corresponding contact: Zainababboud1994@gmail.com

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Abstract. This paper studies the free vibration of simply supported functionally graded beam with material graduation transversally through the thickness using the power-law model. Two finite element models are proposed to calculated the first five frequency parameters of simply supported FG beam. These models are shell and solid models and they are employed using the ANSYS APDL version 17.2. The two models have been verified with the previously published works and found a good agreement with them. Numerical results are presented in graphical forms to study the effects of power-law index (i.e. material distribution) , length-to-thickness ratio, and modulus ratio on the first five frequency parameter of the FG beam. The above mention effects play very important role on the free vibration of the beam. Index of power-law is a parameter that primarily has an effect on the FG beam frequency parameter. It was found that increasing the index of the power law lead to frequency parameter increases when the modules ratio less than one and drops as index of the power law increase when the modules ratio is more one, when the module ratio is equal to one (i.e., pure material), in this case index of power law has no effect. If modulus ratio (E ratio) increases, the frequency parameter increases too, but with the change rate depending on index of power-law and length to height ratio. Also, frequency increase with increasing the length to height ratio at any power-law index.

Keywords: Free Vibration; Functionally Graded Material; Power Law model; FG Beam; Modulus Ratio

List of Nomenclature

Symbols	Definition	Units
nz	Power-law index in thickness direction	---
L/h	Length-to-thickness ratio	---
P(y)	Property at any thickness	---
P_c	Property of the pure ceramic	---
P_m	Property of pure metal	---
λ	Frequency parameter	---
ω	Frequency	rad/sec
I	The moment of inertia of the cross-section of the FG beam	m^4
E_m	Modulus of elasticity of metal	GPa
ρ_m	Density of metal	Kg / m^3
A	The cross section area of the FG beam	m^2
L	Length	m
W	Width	m
h	Thickness	m

1. INTRODUCTION

The selection of material is one of the most important key steps in engineering design. The smart materials with modified properties are used in order to reach the requirement of the machine, system,… etc. Composite materials have a good ability to made a significant change in engineering designs of beams, plates and shells in different composite structures in macro- and nano-scales. FGMs are a class of composite materials which have attracted a lot attention in last decades . FGM consist two or more of two dissimilar material and can be defined as " a class of composites that have continuous variation of material properties from one surface to another and thus eliminate the stress concentration at the interface of the layers found in laminated compositestext "[1]. Functionally graded materials (FGMs), initiated in Japan in 1984 during a space project [2], are increasingly used as structural elements in modern industries such as aerospace structures, turbine blades, rocket engine components, aircrafts, space vehicles and defense industries, electronic, nuclear engineering, aeronautics and biomedical equipment. Due to the importance of the beam structures in the engineering field and increasing use of FGMs, many studies have been accomplished on the bending and vibration problems of FG beams. Sankar [3] introduced an elasticity solution for simply supported FG beams sinusoidal transverse loading assuming an exponential model of Young's modulus and sinusoidal transverse loading. He based on Euler-Bernoulli beam theory to analysis the bending of FG beams.

Chakraborty et al. [4] used the first-order shear deformation theory to developed a new beam element in order to study the thermo-elastic behavior of FG beam. He assumed that the elastic and thermal properties are varying along its thickness as a power-law and exponential models to examine different stress variations. He considered static, wave propagation and free vibration problems in his study to focus on the behavioral difference of FG beams with pure ceramic or a pure metal beams. Goupee and Senthil [5] optimized the natural frequencies of FG structures by tailoring their material distribution for three model problems using a genetic algorithm. The goal of first problem was to find the material distribution at the maximum of first three natural frequencies of FG beam. In the second problem, the goal was minimizing the mass of a FG beam while constraining its natural frequencies to lie outside certain prescribed frequency bands. Finally, the third goal was minimizing the mass of a FG beam by simultaneously optimizing its material distribution and thickness such that the fundamental frequency is greater than a recommended value. Aydogdu and Taskin [6] used Navier type solution method to study the free vibration of simply supported FG beam based on classical beam (CBT) and higher order theories. Young's modulus of beam varies in the thickness direction according to exponential and power models.

Lu et al. [7] used a hybrid state space-based differential quadrature method to find semi-analytical elasticity solutions for thermal deformation and static bending of bi-directional FG beams. Their results showed that the bidirectional functionally graded along the axial direction reduces thermal stresses more than the conventional unidirectional functionally graded materials. Li [8] discussed a new unified approach for analyzing the static and dynamic behaviors of FG beams considering the effects of shear deformation and rotary inertia. He extended the Timoshenko beam theory to treat FG beam as well as layered beams. Xiang and Yang [9] used the differential quadrature method based on Timoshenko beam theory to calculate the free and forced vibration of a thermally pre-stressed, laminated FG beam of variable thickness. The results illustrated that natural frequencies increases and amplitude of vibration decreases when the thicker FGM layers with a smaller volume fraction index are used. Kapuria et al. [10] studied , experimentally and theoretically, the static deflection and free vibration for layered FG beams using third-order zigzag theory and modified rule of mixtures (MROM). This study confirmed the capability of the zigzag theory in accurately modeling the mechanics of layered FG beams with the ceramic content.

Sina et al. [11] employed a new beam theory to study free vibration of FG beams. They used a simple power-law model to describe the variation of beam properties through the thickness. In this work, they used an analytical method to solve the resulting system of ordinary differential equations of the free vibration. The natural frequency result of the new theory is a little different in than the traditional first-order shear deformation beam theory while the mode shapes of the two methods are coincidental. Simsek and Kocaturk [12] based on the Euler–Bernoulli beam theory to derive the system of equations of motion using Lagrange's equations. They studied the dynamic behavior and free vibration characteristics of a simply-supported FG beam under a concentrated moving harmonic load. The results showed that the effects of the different material distribution, velocity of the moving harmonic load, the excitation frequency on the dynamic responses of the FG beam play very important role on the dynamic behavior of the FG beam. Simsek [13] studied dynamic behavior of a functionally graded beam under a moving mass is within the framework of Euler–Bernoulli, Timoshenko and the third-order shear deformation beam theories. Also, Simsek [14] used the Lagrange multiplier method to solve the fundamental frequency of FG beams based on different higher order beam theories.. Huang and Li [15] investigated free vibration of axially functionally graded beams with non-uniform cross-section. They found the natural frequencies of beams with variable flexural rigidity and mass density by transforming the governing equation with varying coefficients to Fredholm integral equations. Wattanasakulpong et al. [16] analyzed free vibration of FG beams using the Ritz method and an improved third order shear deformation theory.

Alshorbagy et al.[17] presented the dynamic characteristics of FG beam with material graduation in transversally through the thickness or axially through the length based on the power law model using the finite element method. They used principle of virtual work under the assumptions of the Euler–Bernoulli beam theory to derive the system of equations of motion. Their model was more effective for replacing the non-uniform geometrical beam with transversally or axially uniform geometrical FG beam. From numerical results, the material distribution, slenderness ratios, and boundary conditions were affected significantly on the dynamic characteristics of the FG beam. Wattanasakulpong et al. [18] investigated the vibration thermal buckling and of FG beams and they found that when the temperature increases towards the critical temperature the fundamental frequency decreases to zero. Thai and Vo [19] studied the bending and free vibration problems of FG beams and proposed a new analytical solutions based on higher-order shear deformation theory. Anandrao et. al.[20] presented free vibration analysis of exponent FG beams for various classical boundary conditions using finite element methods (FEM). They used principle of virtual work to develop two separate elements one based on Timoshenko beam theory and other based on Euler-Bernoulli beam theory. They studied the effect of transverse shear on the mode shapes and natural frequencies for different aspect ratios (length / thickness), volume fraction and boundary conditions. Results shown that transverse shear significantly affects the mode shape and fundamental frequency for lower aspect ratio ratios of FGM beams.

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In this work, two finite element models using ANSYS APDL version17.2 are built to simulate the simply supported FG beam. The first model uses the shell element (SHELL281) while the second model uses solid element (SOLID186). The effects of length to thickness ratio, modulus ratio, mode number and power law index on the frequencies of simply supported FG beam are studied.

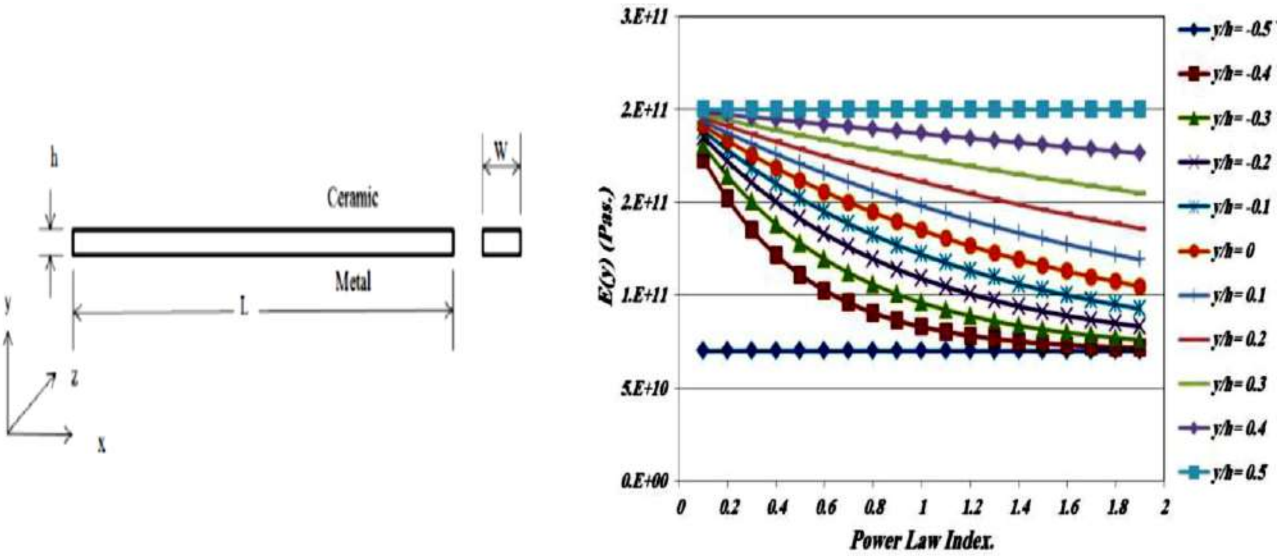
2. METHODOLOGY

2.1. Materials

The dimensions of FG beam with rectangular cross-section are length (L) width (W) thickness (h) as shown in Figure (1-a). The FG beam is constituted by a mixture of two constituents, typically metal and ceramic. A pure ceramic locates at the top surface of the beam while the bottom surface of the beam is a pure metal. The power-law model is used to describe the distribution of mechanical and physical properties along the thickness of beam according and the power-law model is [21] as shown in Figure (1-b).

$$P(y)=(P_c - P_m)\left[\frac{z}{h}+\frac{1}{2}\right]^{nz}+P_m \qquad (1)$$

Where P(y), Pc and Pm represent the property at any thickness, property of the pure ceramic and property of pure metal respectively.



(a) Geometry of Functionally Graded Beam. **(b) Material Distribution.**

Figure 1. Geometry of Functionally Graded Beam.

2.2. Methods

Two finite element models (ANSYS models) are used the current study to simulate the FG beam and these models are:

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1. Shell Model

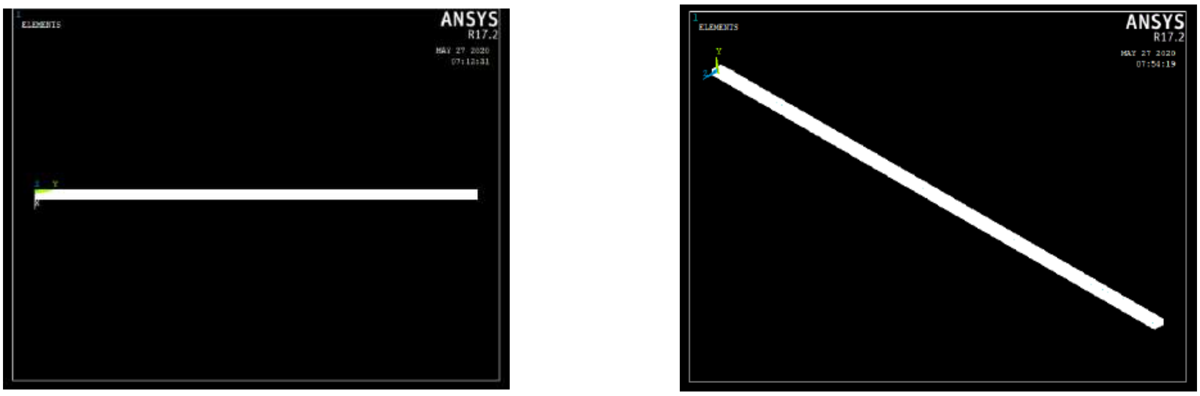
Shell Modeling, two-dimensions model is created by drawing the length and width of FG beam while the thickness of FG beam is represented as a layers have different properties. The shell element (SHELL281) is used to represent the variation in the mechanical and physical properties along the thickness of FG beam. " SHELL281 is well-suited for linear, large rotation, and/or large strain nonlinear applications. Change in shell thickness is accounted for in nonlinear analyses. The element accounts for follower (load stiffness) effects of distributed pressures" [22]. In this work, the thickness of FG beam is divided into ten layers and the number of elements depends length of the FG beam and the convergent criteria of numerical solution and it is about (3250 – 9300) element while the number of nodes is about (10500-30000) node as shown in Figure (2).

2. Solid Model

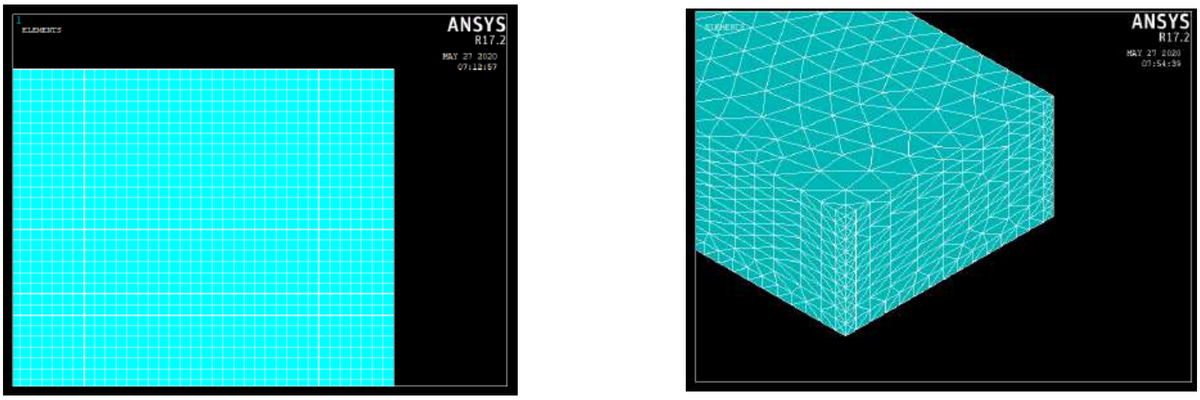
Solid Model is three-dimensions model is created by dividing the thickness of FG beam into ten parts and each parts represents a layer and these ten layers have different properties. The solid element (SOLID186) is used to represent the variation in the mechanical and physical properties along the thickness of FG beam. " SOLID186 is a higher order 3-D 20-node solid element that exhibits quadratic displacement behavior. The element is defined by 20 nodes having three degrees of freedom per node: translations in the nodal x, y, and z directions. The element supports plasticity, hyperelasticity, creep, stress stiffening, large deflection, and large strain capabilities. It also has mixed formulation capability for simulating deformations of nearly incompressible elastoplastic materials, and fully incompressible hyperelastic materials" [22]. The number of elements depends length of the FG beam and the convergent criteria of numerical solution and it is about (28000-285000) element while the number of nodes is about (45000-450000) as shown in Figure (2).

SHELL281 SOLID186

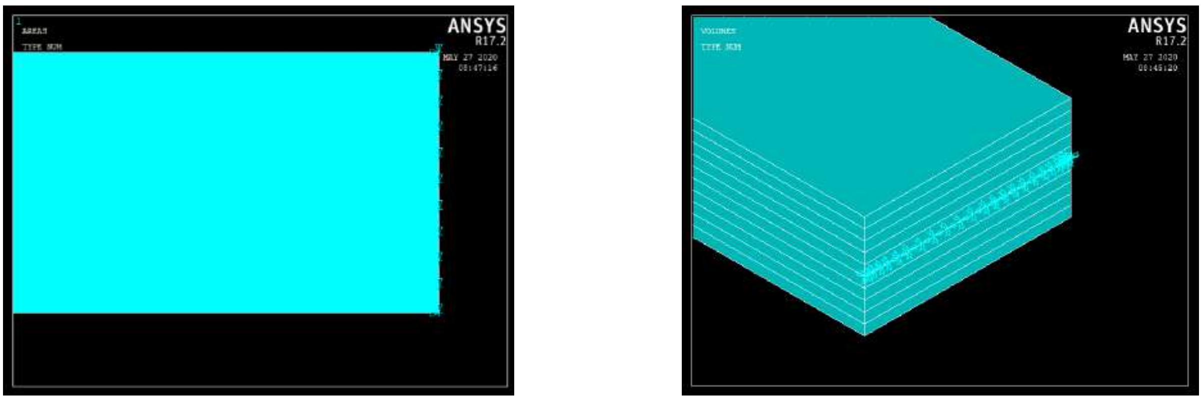
(a) Element Geometry



(b) Meshing of FG Beam



(C) Mesh Shape



(d) Appling Boundary Conditions

Figure 2. Properties and Shapes of Shell and Solids Elements Used in Current Work

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2.2 Validation

In order to validate the two models used in this work, the comparison between the results of Alshorbagy et. al. [17] and Simsek et. al.[14] and the results of the current models were made. Alshorbagy et. al. [17] calculated the frequency parameter for simply supported FG beam. The mechanical properties of FG beam are listed in Table (1).

Table (1). Properties of the Materials Used in This Work [19]

Property	Unit	Metal (Steel)	Ceramic (Alumina (Al2O3))
Modulus of Elasticity (E)	GPas.	210	390
Density(ρ)	Kg/m^3	7800	3960

The dimensions of FG beam are : W (width) = 0.4 m and L (length) = 20 m while the thickness of FG beam is calculated from the length-to-thickness ratio. In this work, the FG beam is considered as a simply supported beam. In order to applied the boundary conditions in shell model, the nodes at the two ends of FG beam are fixed in x, y and z directions. While the nodes at the two ends of neutral plane are fixed in x, y and z directions in the solid model

$$\lambda^2 = \omega L^2 \sqrt{\frac{\rho_m}{E_m}\frac{A}{I}} \qquad (2)$$

The moment of inertia of the cross-section of the FG beam is :

$$I= (bh^3)/12. \qquad (3)$$

Table (2) shows the comparison among the first dimensionless frequency of parameter present work (shell and solid elements) and that calculated by Alshorbagy et. al. [19] and Simsek et. al. [12] when the length – to – thickness ratio is 20.

Table (2). The First Dimensionless Frequency Parameters λ_1 for Different Material Distribution and Different Modulus Ratio(Ec/Em) When (L/h=20) and (ρc/ρm= 1)

Ec/Em	Author	Power – Law Index (K)						
		0.1	0.2	0.5	1	2	5	10
0.25	SHELL281	2.3356	2.4127	2.5695	2.7131	2.8443	2.9487	3.0067
	SOLID186	2.1571	2.2321	2.3863	2.5283	2.6571	2.7654	2.8027
	Alshorbagy et. al.[19]	2.3746	2.4614	2.5979	2.7041	2.8057	2.9302	3.0085
	Simsek et. al.[12]	2.3739	2.4606	--	2.7035	2.8053	--	3.0084
0.5	SHELL281	2.6509	2.6877	2.7685	2.8484	2.9257	2.9977	3.0269

	SOLID186	2.4544	2.4894	2.5678	2.6470	2.7243	2.7928	2.8169
	Alshorbagy et. al.[19]	2.7107	2.7576	2.8363	2.8946	2.9461	3.0110	3.0563
	Simsek et. al.[12]	2.7104	2.7573	--	2.8944	2.9459	--	3.0562
1	SHELL281	3.0647	3.0647	3.0647	3.0647	3.0647	3.0647	3.0647
	SOLID186	2.8442	2.8442	2.8442	2.8442	2.8442	2.8442	2.8442
	Alshorbagy et. al.[19]	3.1400	3.1400	3.1400	3.1400	3.1400	3.1400	3.1400
	Simsek et. al.[12]	3.1399	3.1399	--	3.1399	3.1399	--	3.1399
2	SHELL281	3.587	3.556	3.477	3.387	3.287	3.181	3.133
	SOLID186	3.335	3.306	3.233	3.146	3.045	2.936	2.894
	Alshorbagy et. al.[19]	3.6773	3.6300	3.5296	3.4423	3.3768	3.3196	3.2726
	Simsek et. al.[12]	3.6775	3.6301	--	3.4421	3.3765	--	3.2725
4	SHELL281	4.2298	4.1710	4.0207	3.8371	3.6179	3.3684	3.2497
	SOLID186	3.9361	3.8827	3.7451	3.5726	3.3536	3.0896	2.9819
	Alshorbagy et. al.[19]	4.3366	4.2455	4.0346	3.8241	3.6496	3.5326	3.4549
	Simsek et. al.[12]	4.337	4.2459	--	3.8234	3.6485	--	3.4543

Table (3) shows the same comparison when the length – to – thickness ratio is 100. The maximum and minimum discrepancy percentages between the first dimensionless frequency parameters λ_1 shell and solid models with respect to that of Alshorbagy et. al.[19] (i.e. the summary of Table (2) and (3)) can be seen in Figure (3) and the following points can be noticed.

Table (3): The First Dimensionless Frequency Parameters λ_1 for Different Material Distribution and Different Modulus Ratio(Ec/Em) When (L/h=100) and (ρc/ρm= 1)

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Ec/Em	Author	Power – Law Index (K)						
		0.1	0.2	0.5	1	2	5	10
0.25	SHELL281	2.4832	2.5480	2.6729	2.7765	2.8627	2.9540	3.0126
	SOLID186	2.5040	2.5697	2.6976	2.8059	2.8987	2.9953	3.0533
	Alshorbagy et. al.[19]	2.3798	2.4683	2.6074	2.7159	2.8071	2.9317	3.0100
	Simsek et. al.[12]	2.3752	2.4621	--	2.7053	2.8071	--	3.0100
0.5	SHELL281	2.7604	2.7936	2.8610	2.9192	2.9682	3.0220	3.0582
	SOLID186	2.7766	2.8101	2.8782	2.9376	2.9890	3.0446	3.0807
	Alshorbagy et. al.[19]	2.7121	2.7590	2.8377	2.8961	2.9476	3.0125	3.0578
	Simsek et. al.[12]	2.7117	2.7587	--	2.8960	2.9475	--	3.0578
1	SHELL281	3.1424	3.1424	3.1424	3.1424	3.1424	3.1424	3.1424
	SOLID186	3.1632	3.1632	3.1632	3.1632	3.1632	3.1632	3.1632
	Alshorbagy et. al.[19]	3.1415	3.1415	3.1415	3.1415	3.1415	3.1415	3.1415
	Simsek et. al.[12]	3.1415	3.1415	--	3.1415	3.1415	--	3.1415
2	SHELL281	3.6422	3.6109	3.5403	3.4714	3.4098	3.3409	3.2892
	SOLID186	3.6791	3.6364	3.5644	3.4935	3.4568	3.3830	3.3324
	Alshorbagy et. al.[19]	3.6791	3.6317	3.5313	3.4440	3.3784	3.3213	3.2743
	Simsek et. al.[12]	3.6793	3.6320	--	3.4440	3.3784	--	3.2742
4	SHELL281	4.2656	4.2114	4.0717	3.9268	3.7894	3.6396	3.5254
	SOLID186	4.3450	4.2844	4.1394	3.9842	3.8312	3.6692	3.5577

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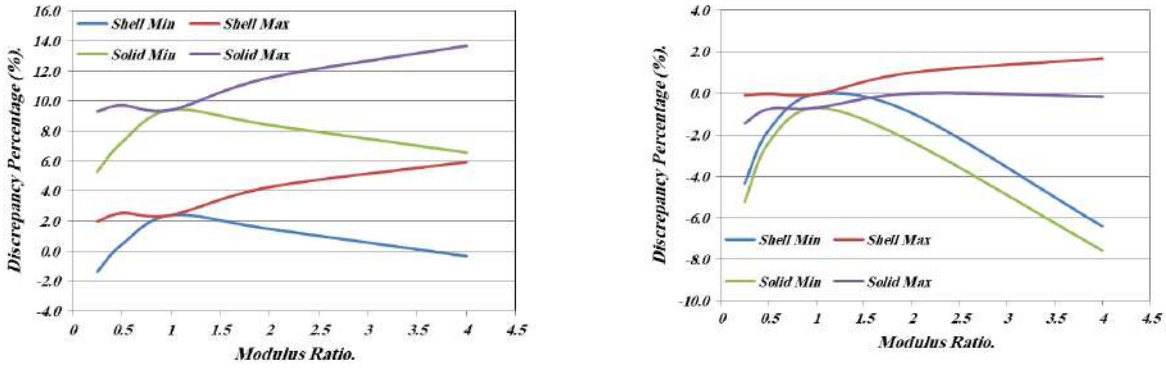
	Alshorbagy et. al.[19]	4.3388	4.2476	4.0366	3.8260	3.5614	3.5343	3.4566
	Simsek et. al.[12]	4.3392	4.2481		3.8259	3.6513		3.4565



(a) L/h=20.

(b) L/h=100.

Figure 3. The Maximum and Minimum Discrepancy Percentages Between the First Dimensionless Frequency Parameters λ_1 Shell and Solid Models with Respect to that of Alshorbagy et. al.[19].

(1) When L/h=20, the results of shell model is closer than the results of solid model. While the results of shell and solid models are very close to each other when L/h=100.

(2) In the solid model , the rang of maximum discrepancy percentage are (9.5 – 14)% when L/h= 20 and (-1.5 – 0)% when L/h= 100. While in the shell model , the rang of maximum discrepancy percentage are (2 –4)% when L/h= 20 and (0 – 1.75)% when L/h= 100
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(3) In the solid model , the rang of minimum discrepancy percentage are (5.5–9)% when L/h= 20 and (-9.5 – 0)% when L/h= 100. While in the shell model , the rang of minimum discrepancy percentage are (-1.5 –-0.5)% when L/h= 20 and (-6.5 –0)% when L/h= 100.

(4) When the modulus ratio increases, the maximum discrepancy percentage increases and the minimum discrepancy percentage decreases.

3. RESULTS AND DISCUSSION

3.1. RESULTS

Several parameters effect on frequency parameter of FG beam. In this work, the following parameters are studied

3.2. DISCUSSION

❖ The Power-Law Index (K)

In the power-law model, the power-law model means distribution of material properties in FG beam. Figure (4) shows the variation of first frequency parameter due to change in power-law index (K) (i.e. material properties) for different (E ratio), length-to-thickness ratio and models. Generally, the frequency parameter increases when the power-law index increases and it tends to be constant at high power-law index when the (E ratio) is less than (1). But when the modulus ratio is larger than (1), the frequency parameter decreases with increasing the power-law index. When the modulus ratio is unity, the power-law index is not effect on the frequency parameter and that's means the beam consists one material only (i.e. not FG beam). The effect of power-law index increases when (E ratio) increases. In other side, the effect of length-to-thickness ratio (L/h) is not appear because the values of first frequency parameters are small. But the values of first frequency parameter, when the length-to-thickness ratio (L/h) is (20), are smaller than that of other length-to-thickness ratios for any modulus ratio and for the two models. Figure (5-6) show the variation of second and third frequency parameters due to change in power-law index (K). When the modulus ratio is (1), the power-law index is not effect on the first five frequency parameters. If the power-law index is smaller than (1), the first five frequency parameters are changed depending on the values of modulus ratio and length-to-thickness ratio (L/h).

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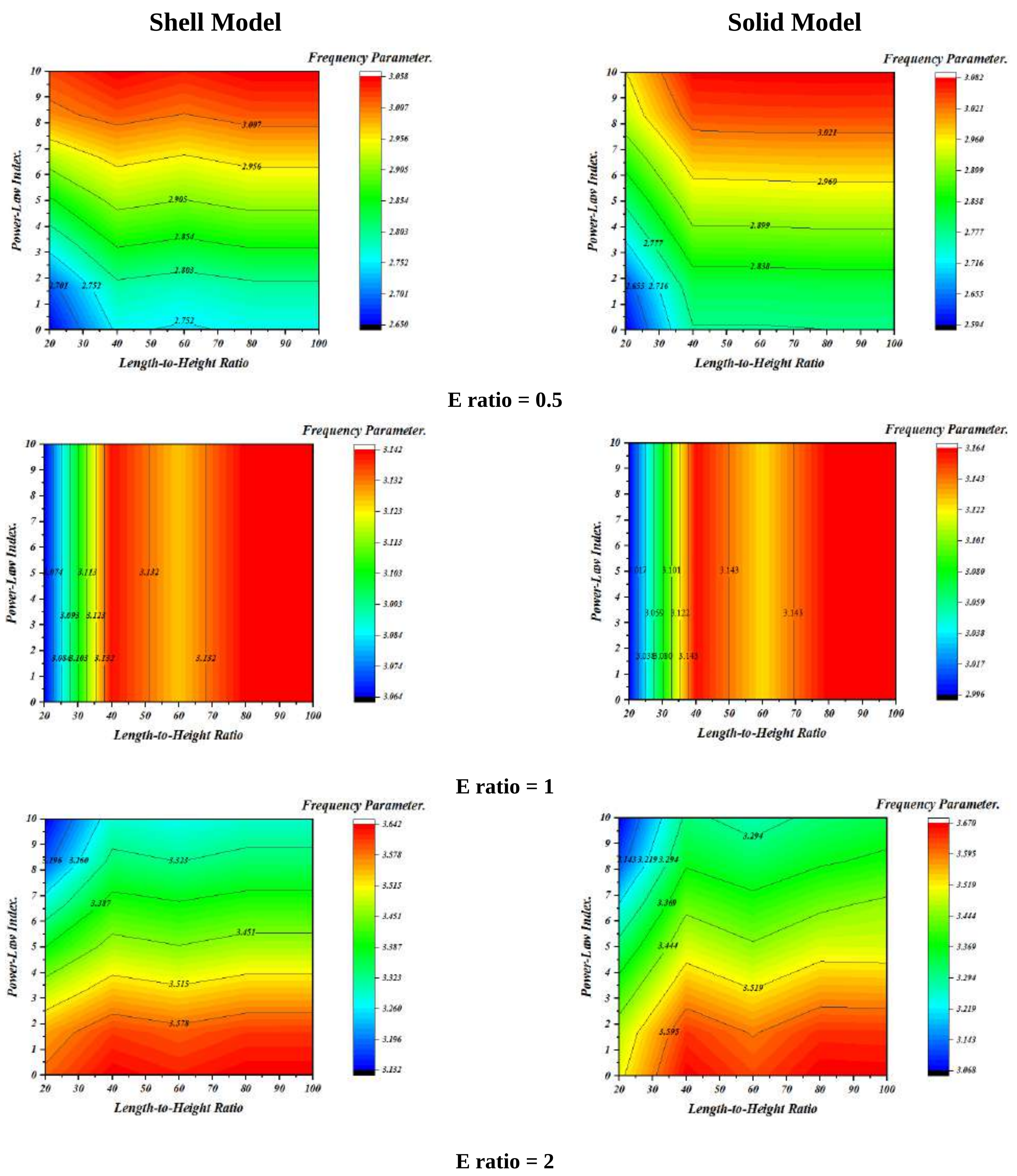

Figure 4. The First Frequency Parameter Variance due to Index of Power-law with Various Length Height Ratio and Modulus Ratio

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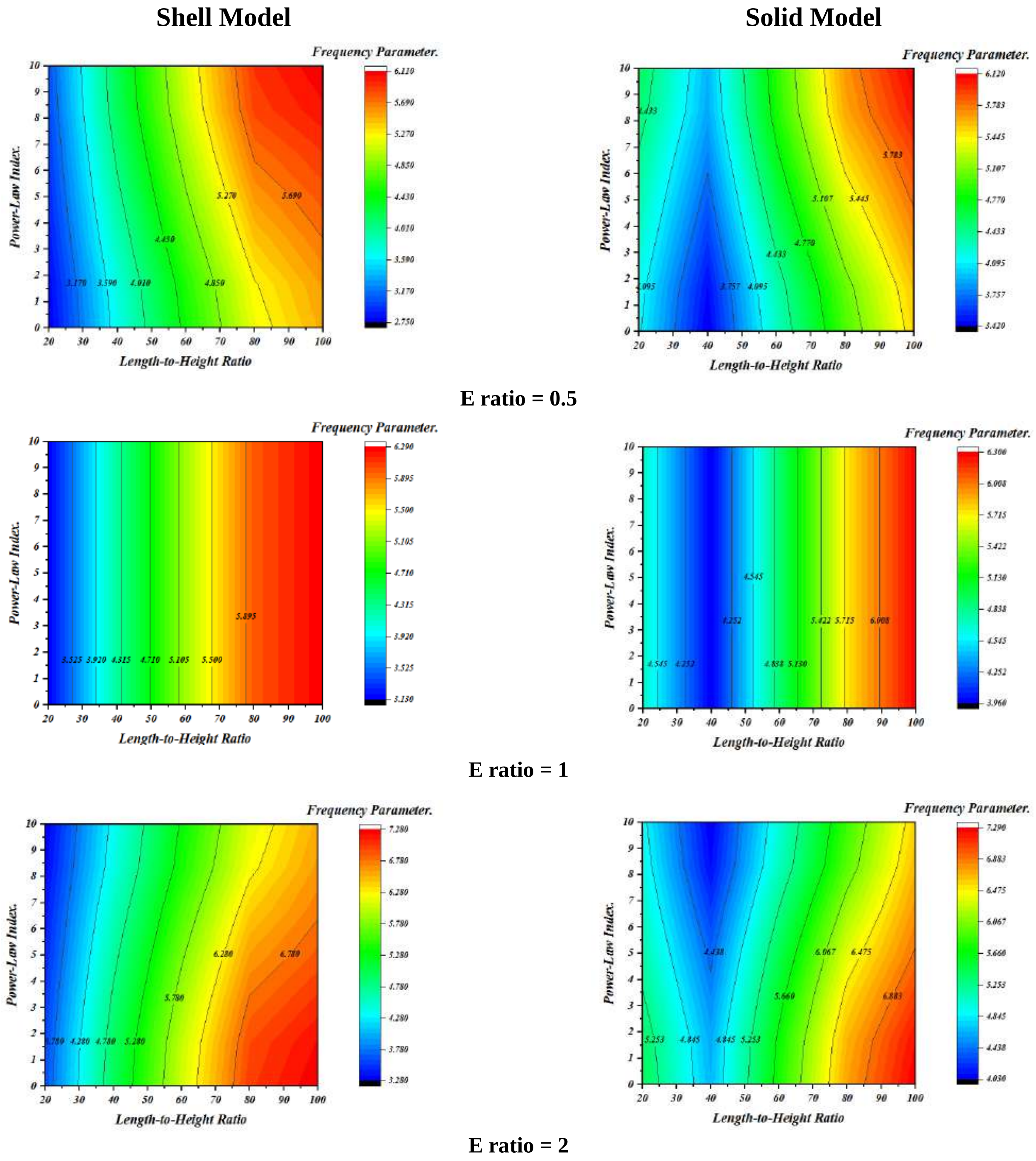

Figure 5. The Second Frequency Parameter Variance due to Index of Power-law with Various Length Height Ratio and Modulus Ratio

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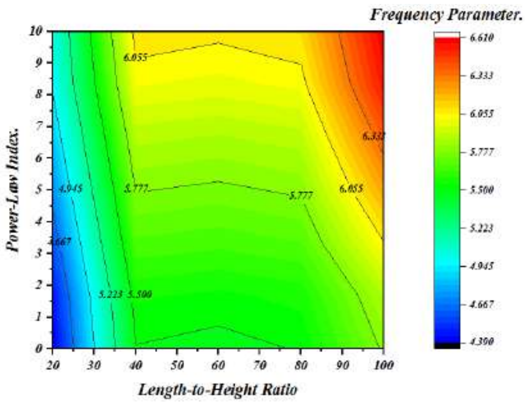
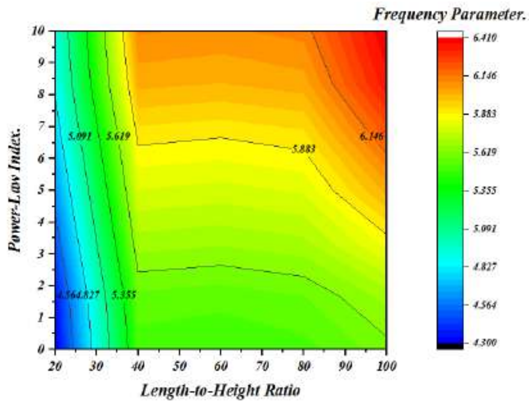
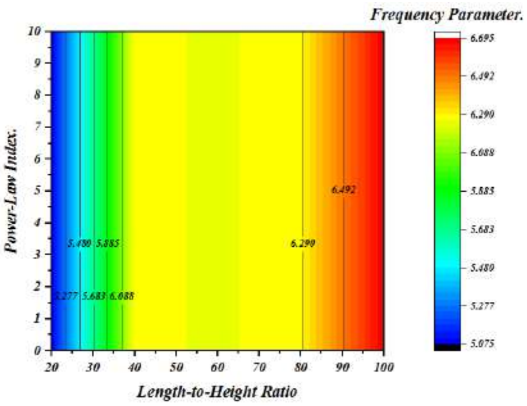
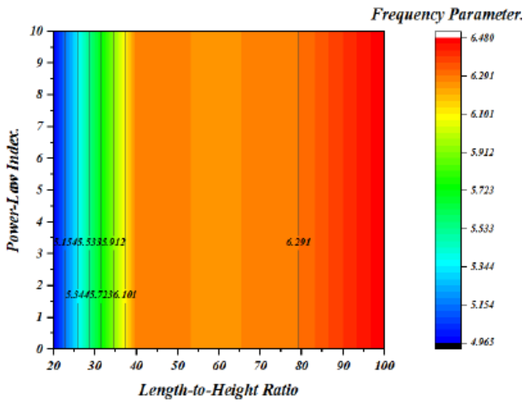
Shell Model **Solid Model**



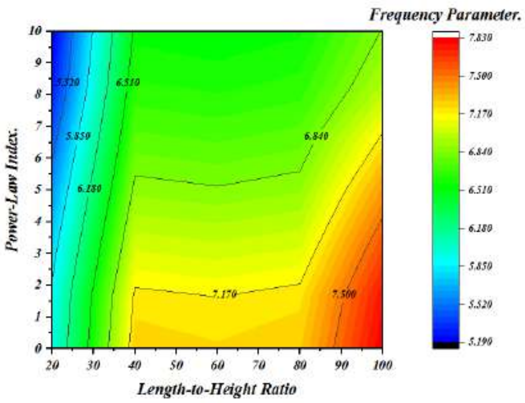


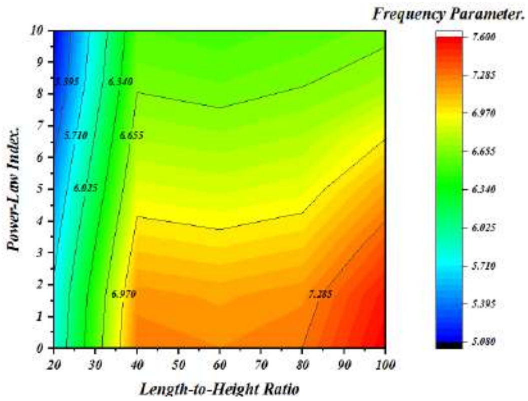
E ratio = 0.5





E ratio = 1





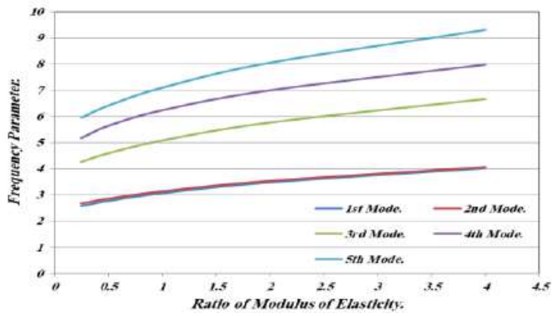
E ratio = 2

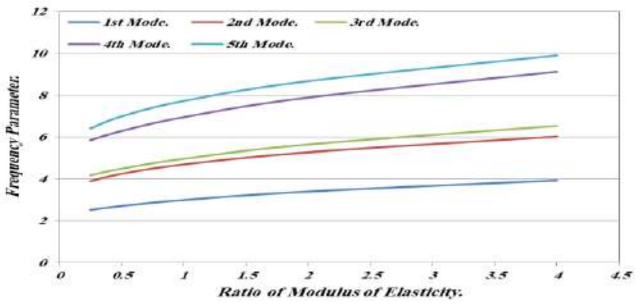
Figure 6. The Third Frequency Parameter Variance due to Index of Power-law with Various Length Height Ratio and Modulus Ratio.

- The Modulus Ratio (E ratio)

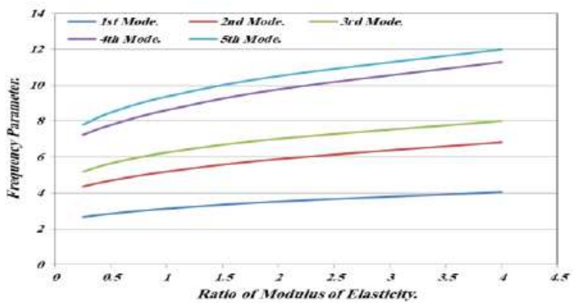
The variation of five frequency parameters due to change in modulus ratio for different length-to-thickness ratio (L/h) and different the power-law index(K) are shown in Figure (7-9). Generally, when the modulus ratio increases, the frequency parameter increases too, but with change rate depending on length-to-thickness ratio (L/h) and power-law index(K) in addition to the mode number. From these Figures, the five frequency parameters converged diverge to each other and the convergence or divergence depends on the model, the power-law index and length-to-thickness ratio (L/h) in additional to modulus ratio For example, the difference between the first and second frequency parameters increases when (a) modulus ratio increases, (b) length-to-thickness ratio (L/h) increases, and (c) power-law index decreases. In Figure (9), the difference between the first and second frequency parameters in shell model is differ than that of solid model and it depends on the length-to-thickness ratio (L/h).

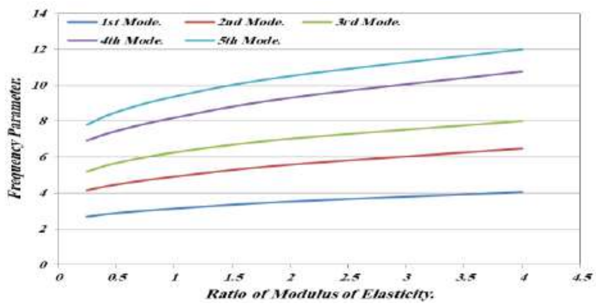
Shell Model **Solid Model**



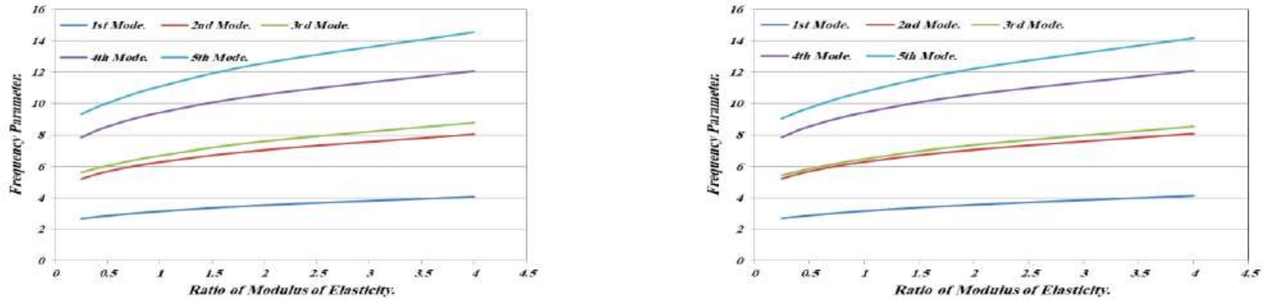


L/h = 20





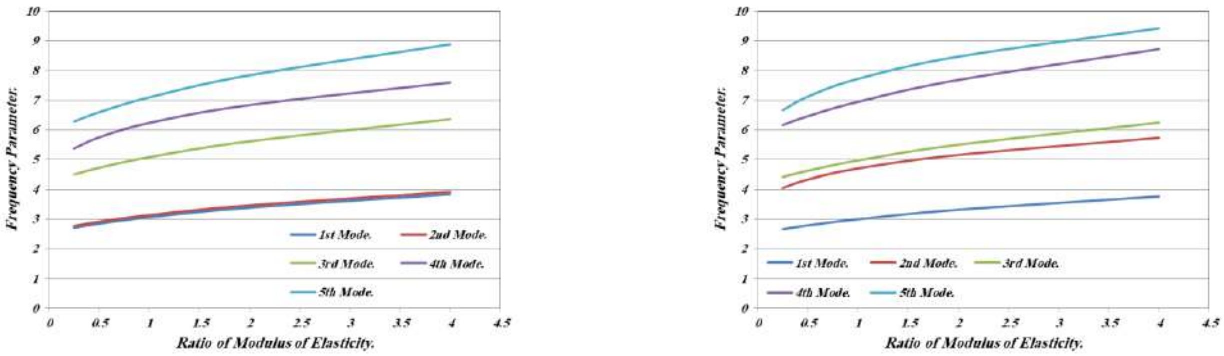
L/h = 60



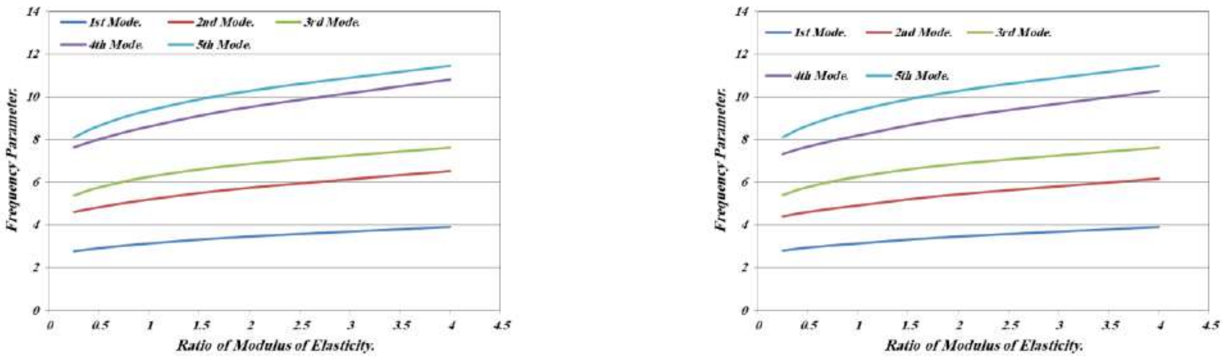
L/h = 100

Figure 7. The Variation of Five Frequency Parameters Due to Change in Modulus Ratio (E ratio) for Different Length-to-Thickness Ratio (L/h) When the Power-Law Index (K) is (0.5)

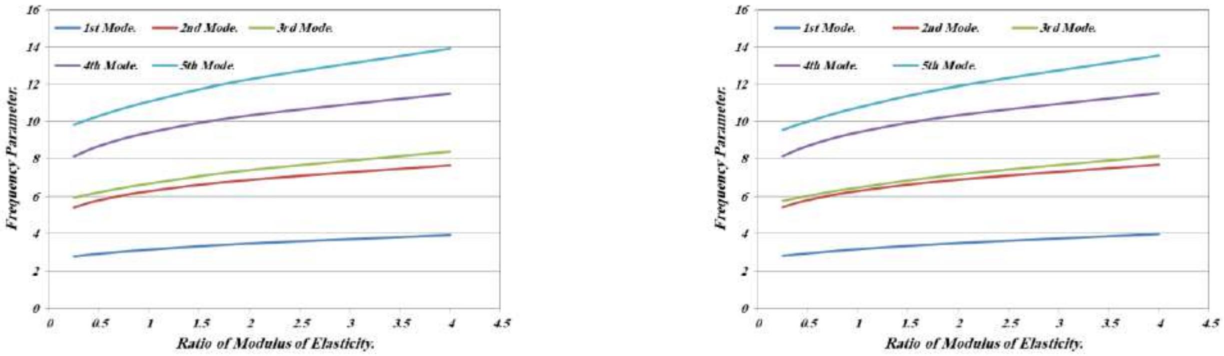
Shell Model **Solid Model**



L/h = 20



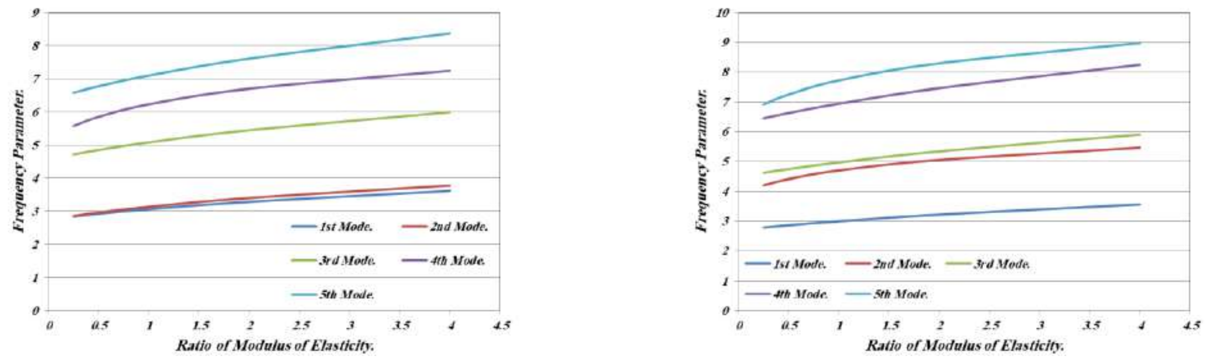
L/h = 60



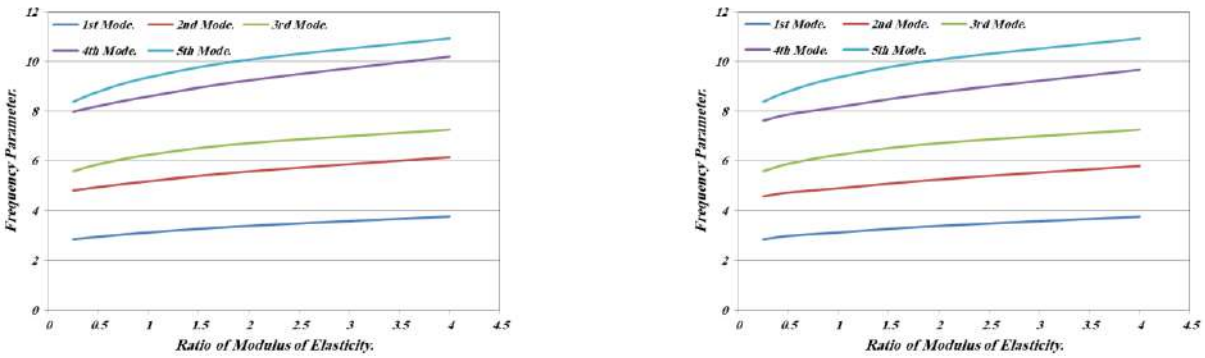
L/h = 100

Figure 8. The Variation of Five Frequency Parameters Due to Change in Modulus Ratio (E ratio) for Different Length-to-Thickness Ratio (L/h) When the Power-Law Index(K) is (1.0)

Shell Model **Solid Model**



L/h = 20



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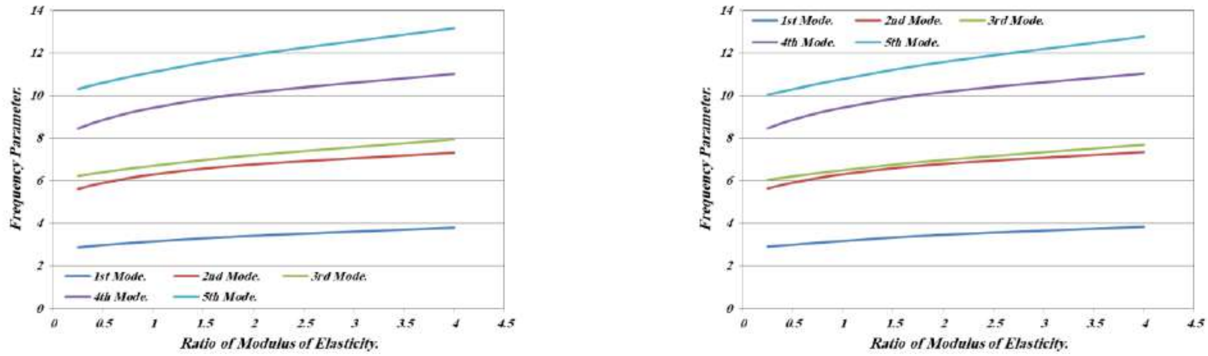
L/h = 60



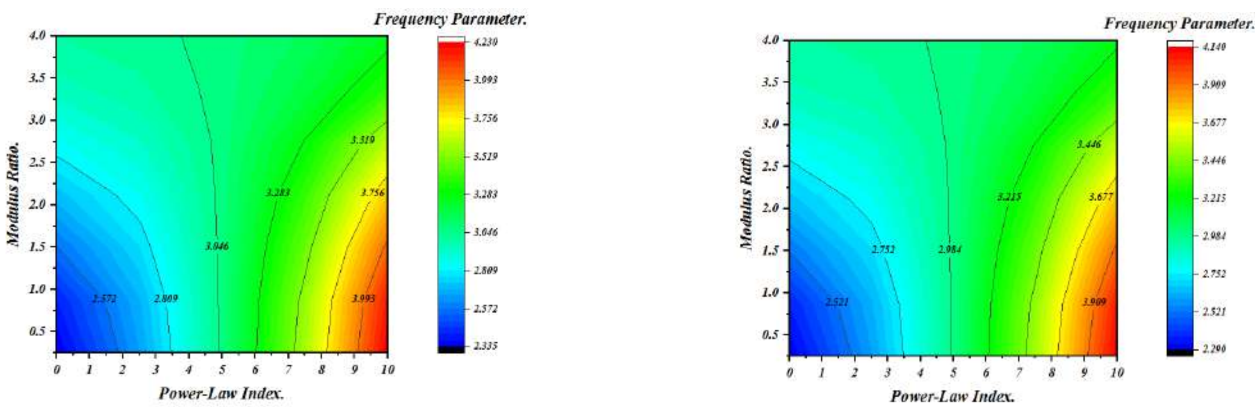
L/h = 100

Figure 9. The Variation of Five Frequency Parameters Due to change in Modulus Ratio (E ratio) for Different Length-to-Thickness Ratio (L/h) When the Power-Law Index(K) is (2.0)

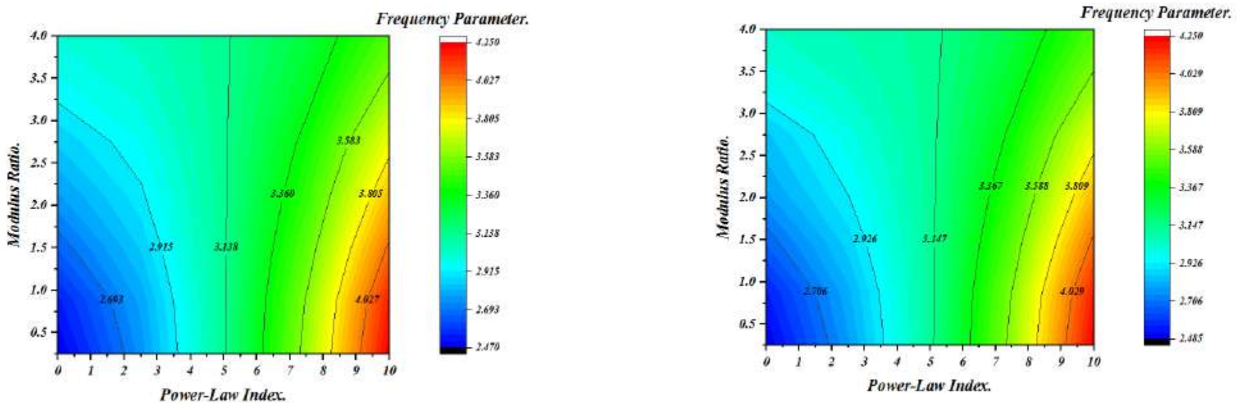
❖ The Length-to-Thickness Ratio (L/h)

Figure (10) shows the variation of first frequency parameter due to change in modulus ratio for different power-law index(K) and length-to-thickness ratio (L/h). It's clear that the variation of the frequency parameter is proportion inversely with power-law index (K) when the modulus ratio is less than (1). While the variation of the frequency parameter is proportion directly with power-law index (K) when the modulus ratio is much than (1). This fact appears in second and third frequency parameters as shown in Figure (11-12). Also, the length-to-thickness ratio (L/h) effects slightly on the frequency parameters for the first and third (see Figure (4,6)) . For the second, the effect of the length-to-thickness ratio (L/h) increases and appears sharply as shown in Figure (5).

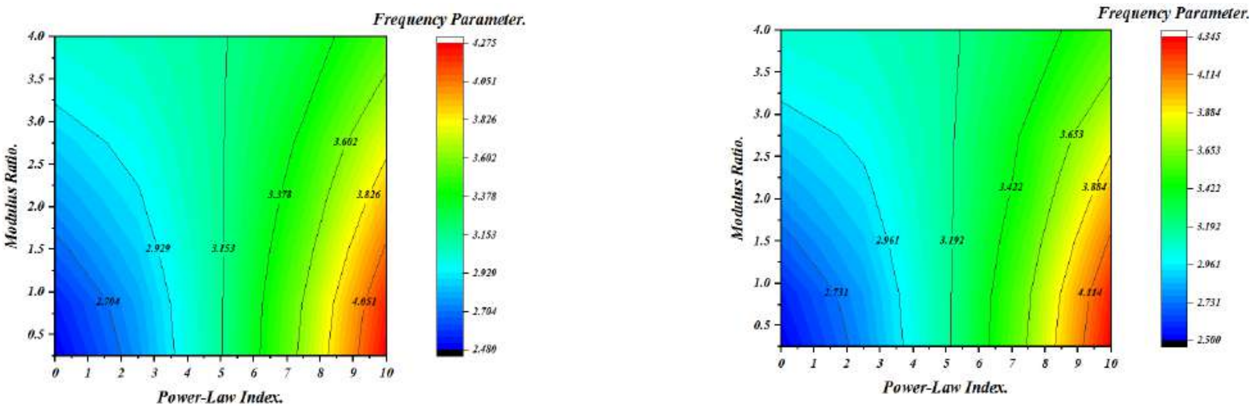
Shell Model **Solid Model**



L/h =20



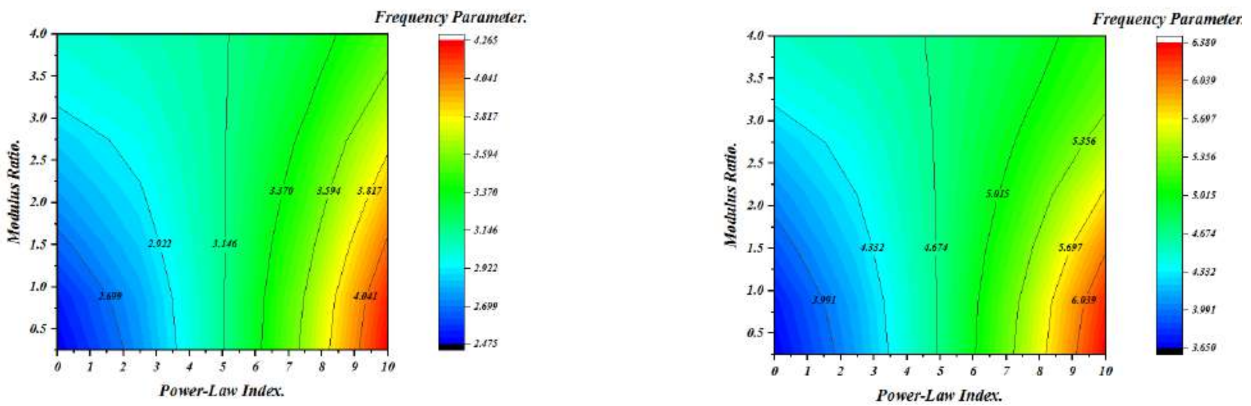
L/h =60



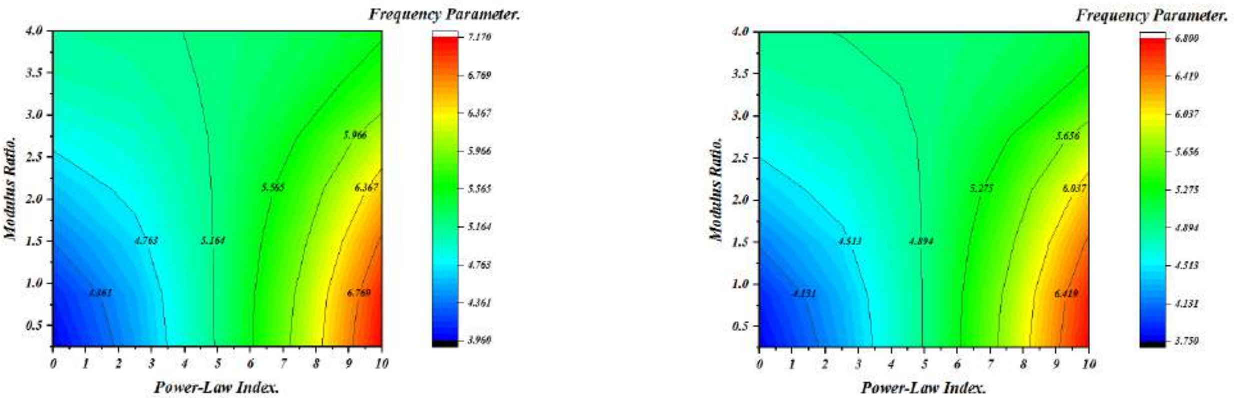
L/h =100

Figure 10. The First Frequency Parameter Variance due to Modulus Ratio (E ratio) for Different Length to Height and Index of Power Law

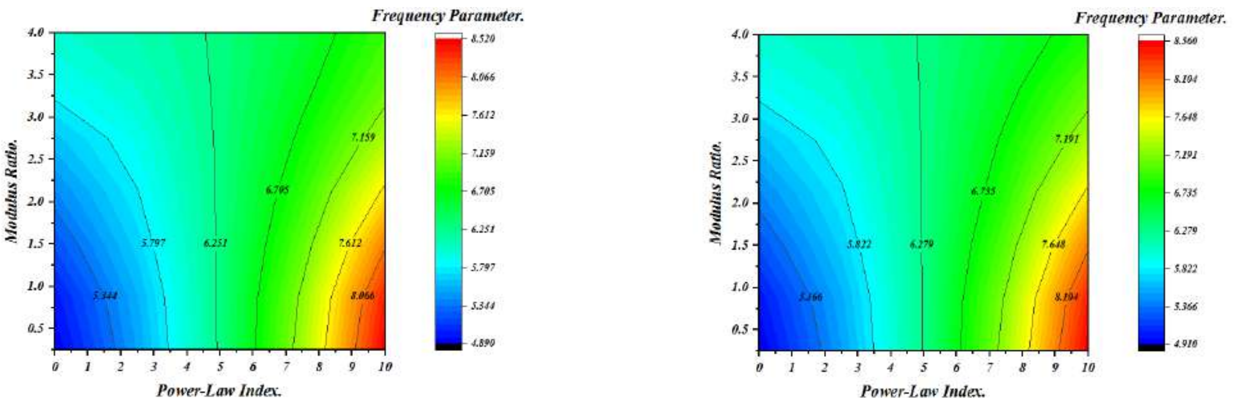
Shell Model **Solid Model**



L/h =20



L/h =60

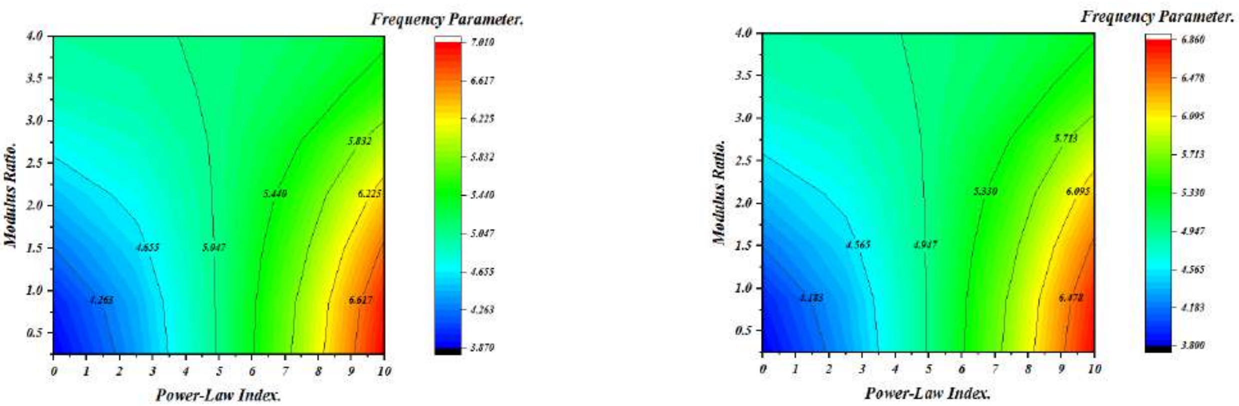


L/h =100

Figure 11. The Second Frequency Parameter Variance due to Modulus Ratio (E ratio) for Different Length to Height and Index of Power Law

Shell Model

Solid Model



L/h =20

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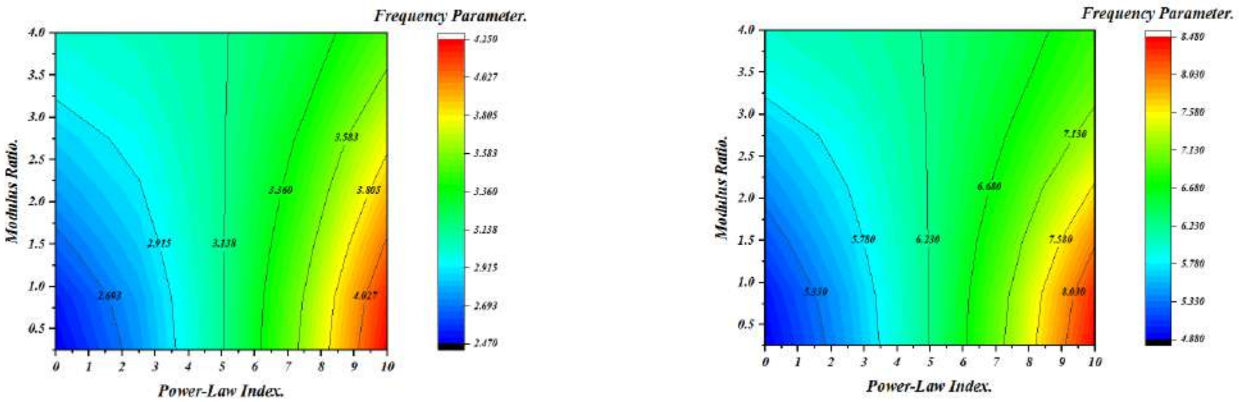
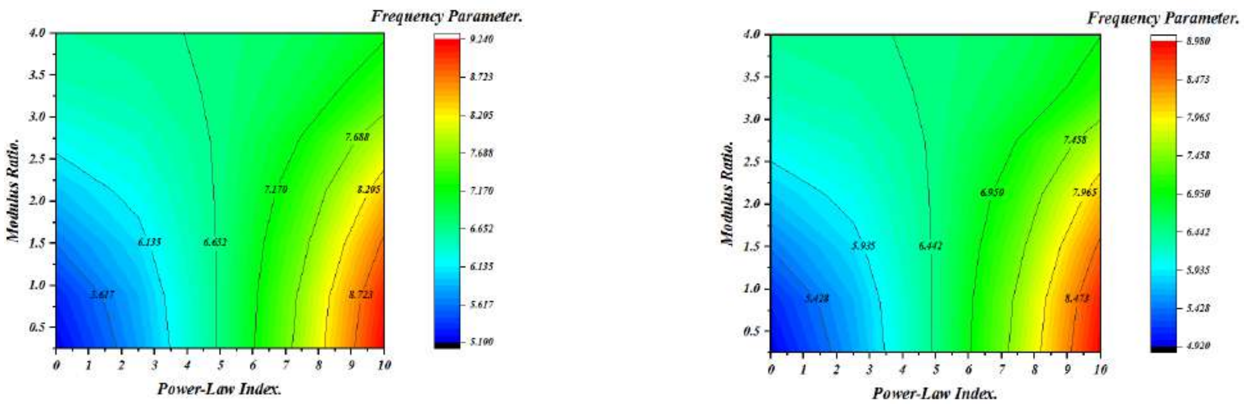


L/h =60



L/h =100

Figure 12. The Third Frequency Parameter Variance due to Modulus Ratio (E ratio) for Different Length to Height and Index of power Law

4. CONCLUSIONS

From the results, the first five frequency parameters of FG simply supported beam are calculated by shell and solid finite element models using ANSYS APDL version 17.2. The power-law model is used to described the material distribution of FG beam. Numerical comparisons are presented to validate the accuracy and convergence of the presented two models. The results show that the frequency parameters depend on;

(1) The power-law index: The frequency parameters for any mode number decrease when the power-law index increases if the modulus ratio is less than (1). While the frequency parameters for any mode number increase when the power-law index increases if the modulus ratio is much than (1).

(2) The length-to-thickness ratio: The length-to-thickness ratio has a slight effects on the first and third frequency parameters and has a sharply effects on the second frequency parameters.

(3) The modulus ratio: The effects of modulus ratio on the frequency parameters increase for high power-law index , high mode number and low length-to-thickness ratio.

(4) the shell model is closer than solid model when they compared with model of Alshorbagy et. al. [19].

Finally, the effects of power-law index, modulus ratio, length-to-thickness ratio and mode number on the frequency parameters of the FG beam can be studied , when the materials are distributed along the thickness and /or width of beam.

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